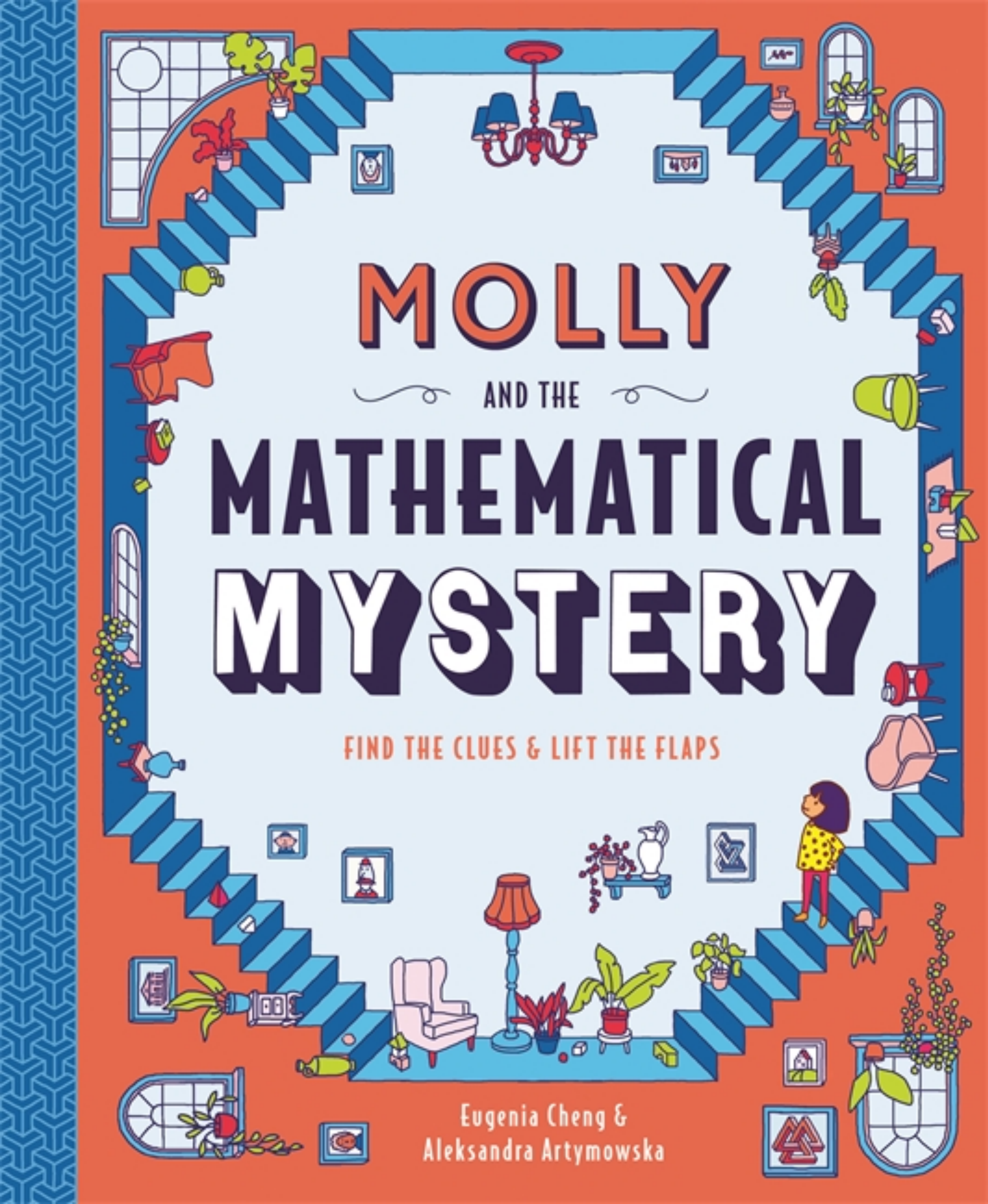

MOLLY AND THE MATHEMATICAL MYSTERY

FIND THE CLUES & LIFT THE FLAPS

Eugenia Cheng &
Aleksandra Artymowska

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To my nephews,
Liam and Jack
– E.C.

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MOLLY AND THE MATHEMATICAL MYSTERY

When you think about maths, what do you think about? Do you think it's all just numbers and equations? Then think again...

Maths is *really* about exploring ideas and using our imagination. It's a way of making sense of the world – or of making seemingly impossible things possible!

Molly is a curious explorer, and she is about to go on a fantastic adventure by following a trail of mysterious clues. She'll need to be imaginative – and she just might need some help. So, are you feeling adventurous? Then step inside!

THE ADVENTURE BEGINS...

This is Molly. She's an ordinary girl with an ordinary bedroom. But look, what's that on the floor?

It's a note with Molly's name on it. How strange! She opens it up and starts to read. It's an invitation to do something EXTRAordinary. Can you work out what the note is asking her to do? Let's give it a try...

Molly opens the window and imagines pushing her room right through it. She hears a whooshing sound. It gets louder and louder. Suddenly Molly is swept right up in the sound and everything becomes a blur...

Push open the window to follow Molly on her adventure.

Turning inside out

Can you imagine turning a room inside out like turning a giant sock inside out?

Think about holding a sock (or even find a real sock to experiment with!). To turn the sock inside out you have to push the material through the round opening. Now imagine the opening is a window, and the material is your room. Turning your bedroom inside out doesn't seem so impossible anymore... at least in your imagination! This is what maths is all about.

How many socks can you find in Molly's bedroom? Can you match three pairs?

IT'S ALL INSIDE OUT

Something peculiar is going on. All of Molly's bedroom furniture is stuck to the outside of her house. It's her bedroom... turned inside out! Molly has done it!

She seems to have turned some other things inside out as well. The tools are all hanging on the outside of the shed, and look what's happened to the pond. And what's that weird pillar of earth?

A white rabbit darts up the pillar and Molly realises it's an inside-out rabbit hole! She follows the rabbit and starts climbing. Will that note help? And who left it there?

Inside-out maths

When you turn something inside out, it's like looking into a peculiar mirror: the inside becomes the outside, up becomes down. Everything is opposite!

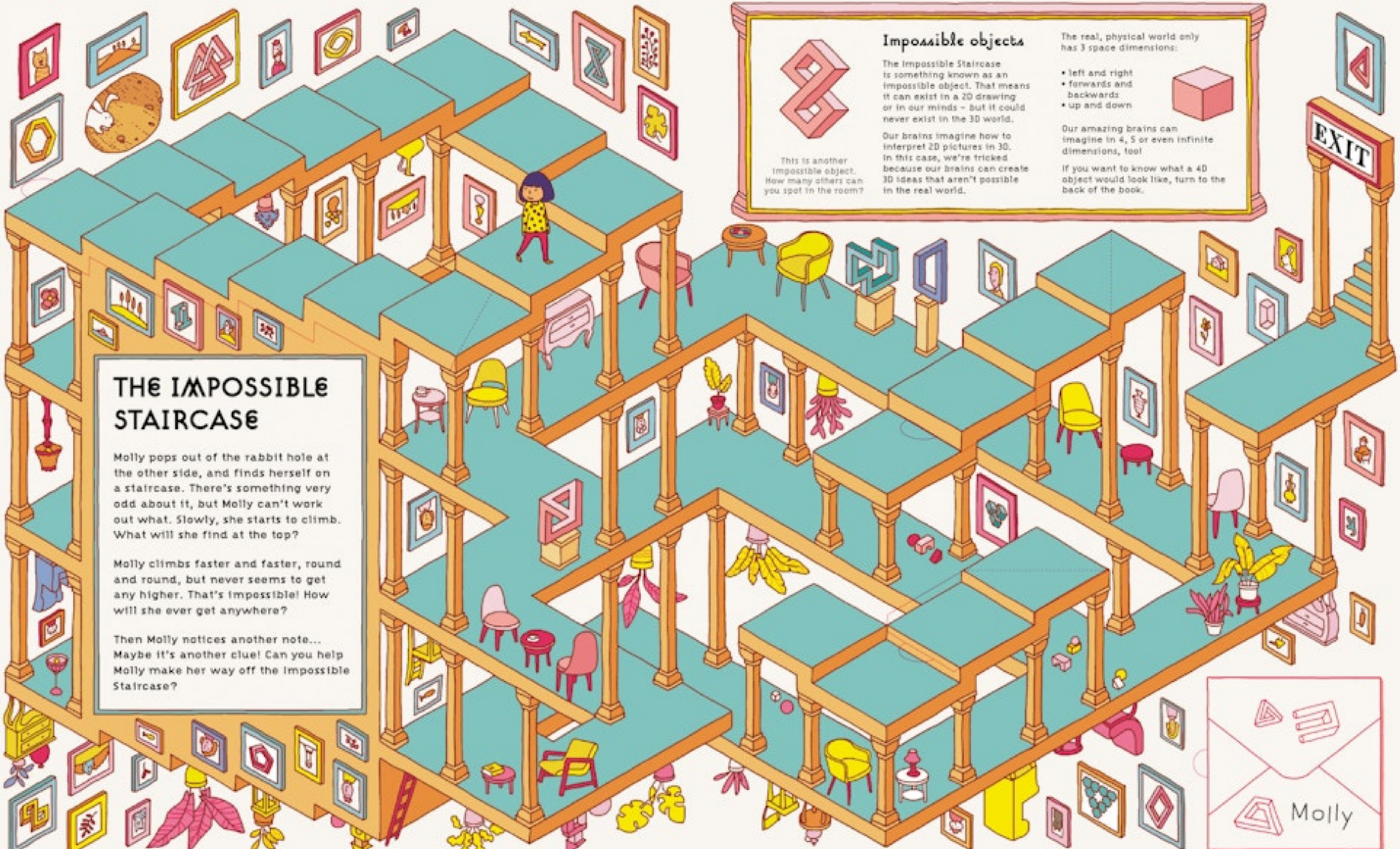
There are many different opposites in maths. They are sometimes called inverses. For example, the inverse of multiplication is division. The inverse of adding is subtracting.

If you invert something, you can always invert it again to get back to where you started. Just like the negative of a negative number is positive:

$-(-1) = 1$

That should mean Molly can get back to normal again. But first she wants to explore this strange place!

What other inside-out things can you see on this page?



THE IMPOSSIBLE STAIRCASE

Molly pops out of the rabbit hole at the other side, and finds herself on a staircase. There's something very odd about it, but Molly can't work out what. Slowly, she starts to climb. What will she find at the top?

Molly climbs faster and faster, round and round, but never seems to get any higher. That's impossible! How will she ever get anywhere?

Then Molly notices another note... Maybe it's another clue! Can you help Molly make her way off the Impossible Staircase?

Impossible objects

This is another impossible object. How many others can you spot in the room?

The Impossible Staircase is something known as an impossible object. That means it can exist in a 2D drawing or in our minds – but it could never exist in the 3D world.

Our brains imagine how to interpret 2D pictures in 3D. In this case, we're tricked because our brains can create 3D ideas that aren't possible in the real world.

The real, physical world only has 3 space dimensions:

- left and right
- forwards and backwards
- up and down

Our amazing brains can imagine in 4, 5 or even infinite dimensions, too!

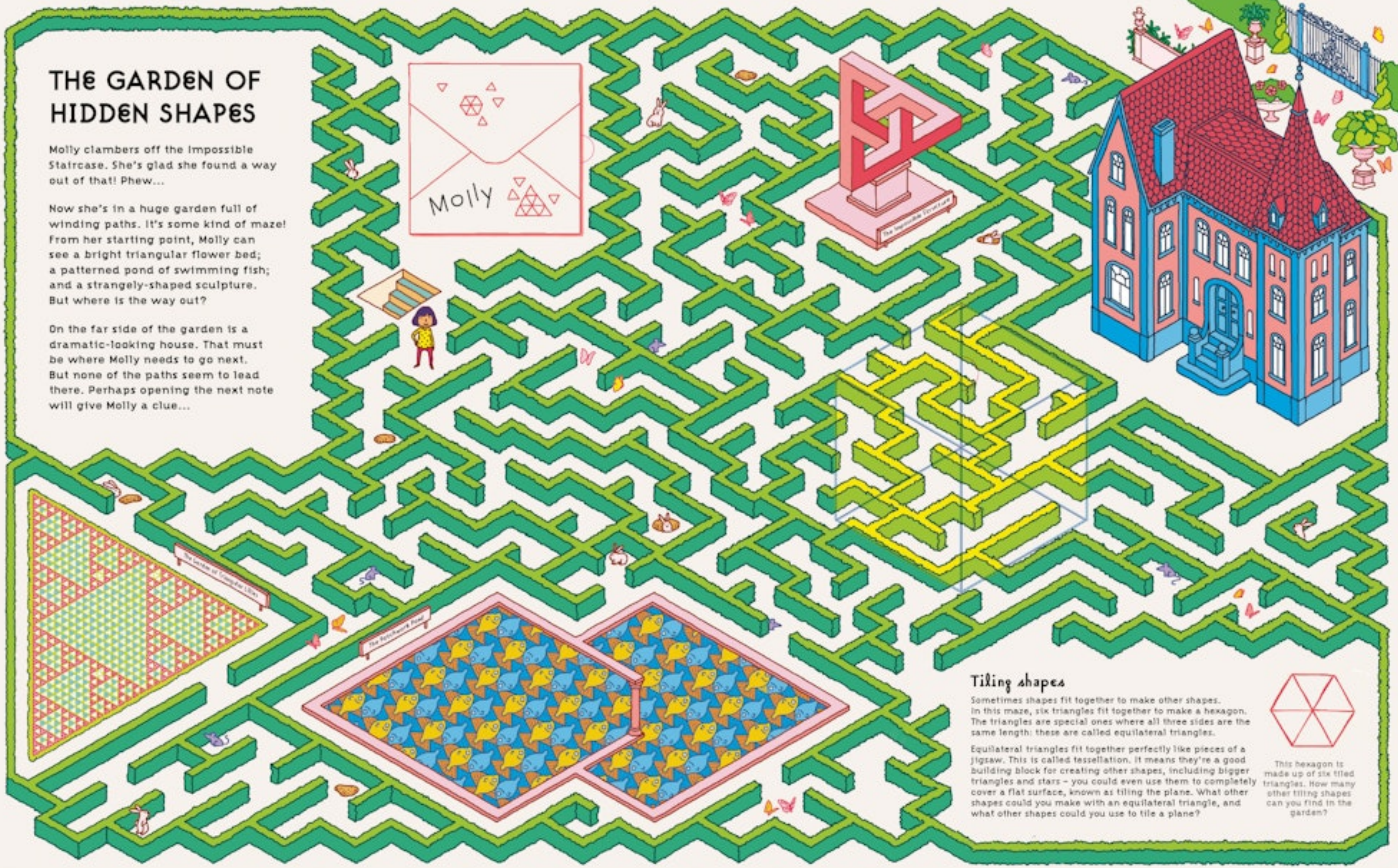
If you want to know what a 4D object would look like, turn to the back of the book.

THE GARDEN OF HIDDEN SHAPES

Molly clambers off the Impossible Staircase. She's glad she found a way out of that! Phew...

Now she's in a huge garden full of winding paths. It's some kind of maze! From her starting point, Molly can see a bright triangular flower bed; a patterned pond of swimming fish; and a strangely-shaped sculpture. But where is the way out?

On the far side of the garden is a dramatic-looking house. That must be where Molly needs to go next. But none of the paths seem to lead there. Perhaps opening the next note will give Molly a clue...



Tiling shapes

Sometimes shapes fit together to make other shapes. In this maze, six triangles fit together to make a hexagon. The triangles are special ones where all three sides are the same length: these are called equilateral triangles.

Equilateral triangles fit together perfectly like pieces of a jigsaw. This is called tessellation. It means they're a good building block for creating other shapes, including bigger triangles and stars – you could even use them to completely cover a flat surface, known as tiling the plane. What other shapes could you make with an equilateral triangle, and what other shapes could you use to tile a plane?

This hexagon is made up of six tiled triangles. How many other tiling shapes can you find in the garden?

THE HALL OF ENDLESS DOORS

Molly makes her way through the maze and reaches the front door of the house. She steps inside and looks around in awe. In front of her is a grand staircase, and a dizzying hallway full of doors. Where could they all lead?

The doors all have intricate patterns on them, but there's no sign of what's behind each one. Look, there's another note. Will it help Molly find the way out? Make sure you don't open a never-ending door, or you'll end up in a never-ending room with a never-ending hallway!

Self-symmetry

The doors in this hallway look like they go on forever! Each door pattern has a smaller door inside it, with an even smaller door inside that, and so on... They're a bit like nesting dolls. We could keep zooming in forever... providing the doors keep going forever.

All the smaller doors look identical to the biggest door. This type of symmetry is called self-symmetry.

In maths, something that keeps going forever is called infinite. Let's imagine an infinite set of dolls. You could keep opening them forever without running out of dolls, but they'd get so tiny it would be very hard to see them!

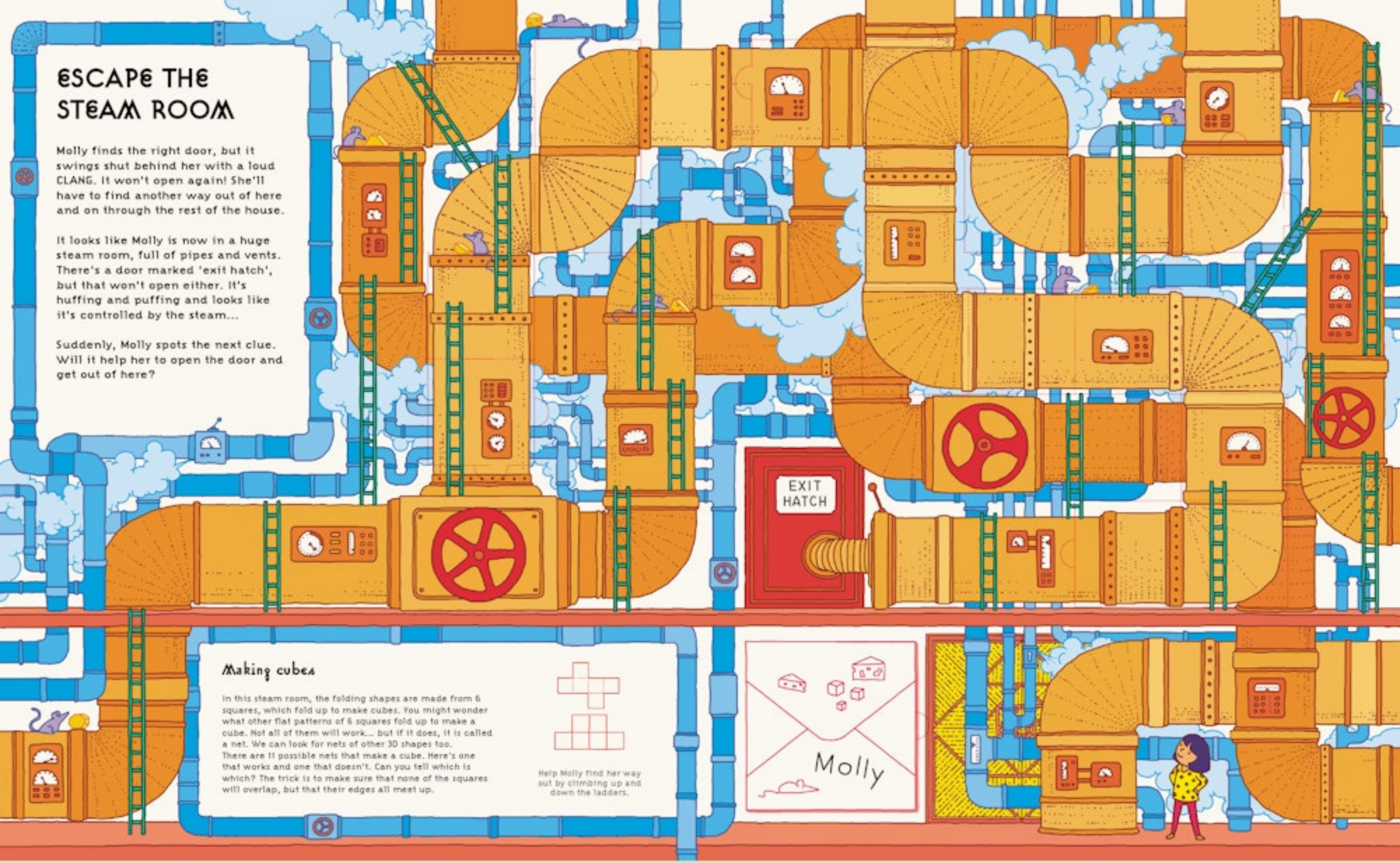
How many nesting dolls can you find hidden in the scene? Can you match up three sets of five dolls?

ESCAPE THE STEAM ROOM

Molly finds the right door, but it swings shut behind her with a loud CLANG. It won't open again! She'll have to find another way out of here and on through the rest of the house.

It looks like Molly is now in a huge steam room, full of pipes and vents. There's a door marked 'exit hatch', but that won't open either. It's huffing and puffing and looks like it's controlled by the steam...

Suddenly, Molly spots the next clue. Will it help her to open the door and get out of here?



Making cubes

In this steam room, the folding shapes are made from 6 squares, which fold up to make cubes. You might wonder what other flat patterns of 6 squares fold up to make a cube. Not all of them will work... but if it does, it is called a net. We can look for nets of other 3D shapes too. There are 11 possible nets that make a cube. Here's one that works and one that doesn't. Can you tell which is which? The trick is to make sure that none of the squares will overlap, but that their edges all meet up.

Help Molly find her way out by climbing up and down the ladders.

CARPET WEAVING

The steam-controlled door bursts open... is Molly nearly home? The next room she enters is filled with wonderfully wacky tapestries. There are carpets all over the floor, the walls and even on the ceiling!

Some of the carpets have curious designs. They look like some of the places Molly has been to before. Do you recognise any of them? Others are decorated with shapes and patterns.

The biggest carpets have been cut into strips. Molly wonders why someone would do that. There's a note here too. Will it help her again?

Latin squares

Take a look at the hanging carpets above. The pattern only shows each shade of green or pink once in each row and once in each column. Both carpets do this, just the colours are switched. This is called a Latin square.

When you weave the strips together with an over-under technique, you will either reveal a pink pattern or a green one. This is because each of the strips alternates pinks and greens, but one carpet starts with green in the top left and the other starts with pink. Here is another 4 x 4 Latin square which uses shapes as well as colours:

Look closely at the other hanging carpets on the wall. Can you find somewhere Molly hasn't visited yet? Perhaps that's where she is headed next!

MIXED-UP LIBRARY

Molly squeezes through the exit door and finds herself in a gigantic library. Polished wooden bookcases are bursting with books in all shapes and sizes.

Some of the books are tiny, but some are bigger than Molly herself! Molly notices a pile of five books that have fallen on the floor. They look like they're a series. She loves books so she wants to start reading them, but she doesn't know what order they're supposed to be in.

Open the next note to find out how to work it out... and see if it reveals how to enter the next room.

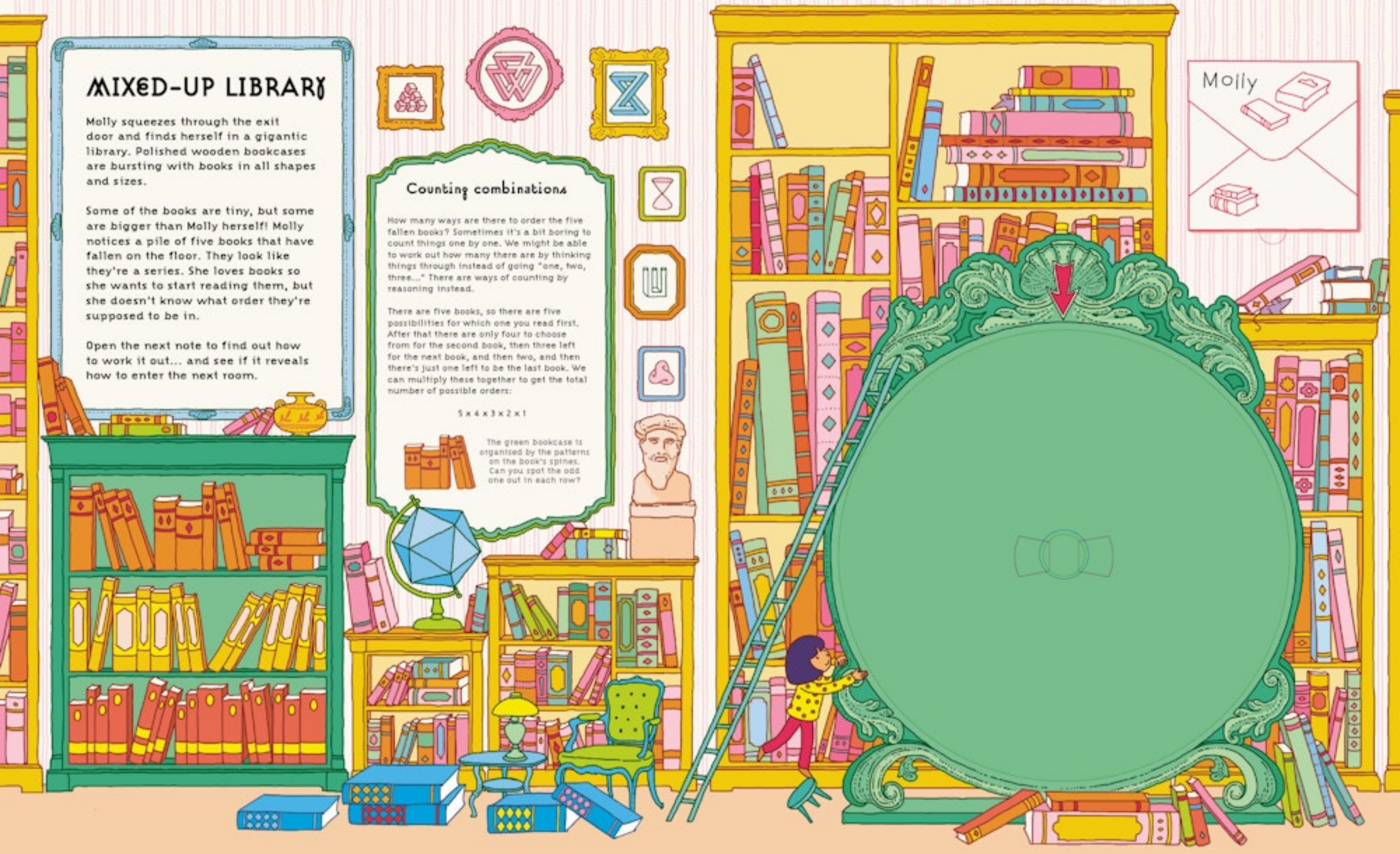
Counting combinations

How many ways are there to order the five fallen books? Sometimes it's a bit boring to count things one by one. We might be able to work out how many there are by thinking things through instead of going "one, two, three..." There are ways of counting by reasoning instead.

There are five books, so there are five possibilities for which one you read first. After that there are only four to choose from for the second book, then three left for the next book, and then two, and then there's just one left to be the last book. We can multiply these together to get the total number of possible orders:

$$5 \times 4 \times 3 \times 2 \times 1$$

The green bookcase is organised by the patterns on the book's spines. Can you spot the odd one out in each row?



SYMMETRY GARDEN

The bookcase creaks open to reveal a luscious green garden. Molly has made her way right through the house!

The garden is full of colourful butterflies flitting from flower to flower. There's a gate leading out of the garden, but it won't open. Will the next letter explain how the gate works?

Let's help Molly to complete her next task and open up the gates. Who knows where she will go next?

Symmetry

There are several types of symmetry. One kind is where you can fold something in half and it's the same on both sides – this is called reflectional symmetry.

Another kind is where you can turn something round and it still looks the same, like a windmill – this is called rotational symmetry.

There's also a kind where you can move something sideways or in another straight line direction and it looks the same, like a fence or the pattern of a brick wall – this is called translational symmetry.

Can you find all three types of symmetry in the garden?

THE FRACTAL ORCHARD

On the other side of the gates is a strange-looking orchard. All the trees are curiously regular, with each branch splitting in two again and again and again...

Molly can't see a way out of here, and the whole orchard is surrounded by a wall. She's good at climbing trees, but the tallest tree isn't high enough to help her get over the wall.

There's yet another clue! Will it help her to climb out?
And where are these curious clues coming from anyway?

Fractals

These trees grow quickly! As you look up, the number of branches is multiplied by two at each level. When you repeatedly multiply by the same number like this it's called an exponential, and the numbers grow very fast even when you're starting with a small number like 2.

Starting at the trunk, the numbers go like this: 1, 2, 4, 8...

Using this pattern, can you work out how many levels you need to get 32 branches at the top, without counting the branches one by one?

These trees are a bit like the doors in the infinite corridor: if they kept branching forever they would have self symmetry, because at each branching point the tree would have two smaller copies of itself.

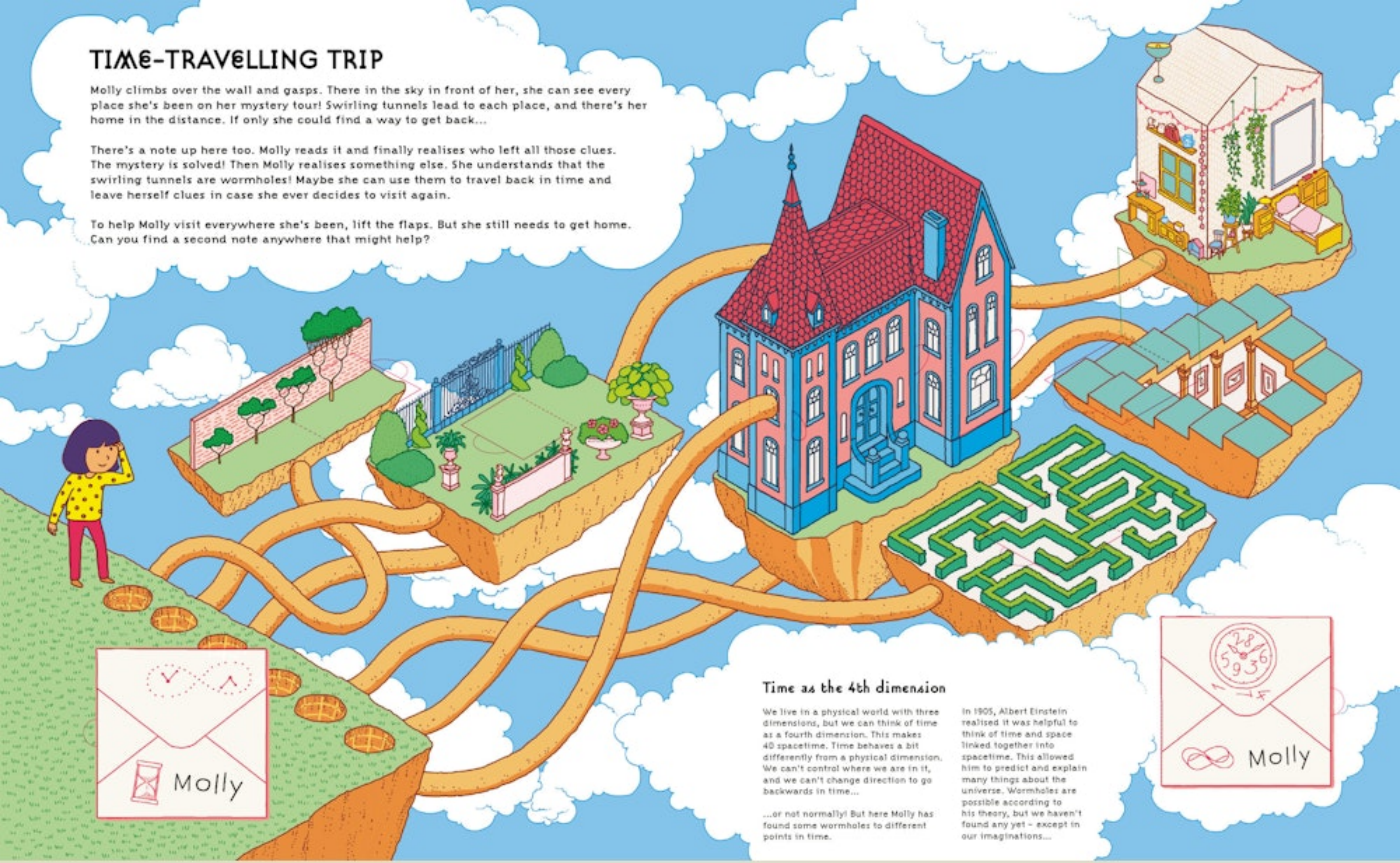
Can you find a watering can in the scene to make the biggest fractal tree grow?

TIME-TRAVELLING TRIP

Molly climbs over the wall and gasps. There in the sky in front of her, she can see every place she's been on her mystery tour! Swirling tunnels lead to each place, and there's her home in the distance. If only she could find a way to get back...

There's a note up here too. Molly reads it and finally realises who left all those clues. The mystery is solved! Then Molly realises something else. She understands that the swirling tunnels are wormholes! Maybe she can use them to travel back in time and leave herself clues in case she ever decides to visit again.

To help Molly visit everywhere she's been, lift the flaps. But she still needs to get home. Can you find a second note anywhere that might help?



Time as the 4th dimension

We live in a physical world with three dimensions, but we can think of time as a fourth dimension. This makes 4D spacetime. Time behaves a bit differently from a physical dimension. We can't control where we are in it, and we can't change direction to go backwards in time...

...or not normally! But here Molly has found some wormholes to different points in time.

In 1905, Albert Einstein realised it was helpful to think of time and space linked together into spacetime. This allowed him to predict and explain many things about the universe. Wormholes are possible according to his theory, but we haven't found any yet – except in our imaginations...

THERE'S NO PLACE LIKE HOME

Molly wakes up on top of her bed. Everything looks normal – her bedroom is the right way round, and everything's just as she left it. Nothing seems to be inside out now. But Molly's head is full of patterns, shapes and impossible objects.

"I've had such a strange dream," she says. But then her eyes fall on a note on the floor. She doesn't remember leaving that there...

Does that mean her implausible adventure could actually have happened? And can you see anything that's still inside out?

MORE MATHS

Now you've finished your adventure, maybe you're feeling more curious about the things you learned on the way. Browse these pages to find out more about maths.

Maths isn't just about numbers. It's about understanding the world by finding similarities between different things. Instead of "spot the difference", we can play "spot the similarity"!

Here are some examples of mathematical similarities we can spot in every day life. What do you notice?

Abstraction

When you spot similarities, you're seeing through details on the surface and thinking about BIG ideas. This is where maths comes from! This is also how Molly begins her journey.

Seeing past the surface is called abstraction and takes us from the world of things we can touch to the world of ideas in our imagination. If you eat ice cream, it's a real object. But if you're hungry and dreaming of eating ice cream, it's an idea. It's abstract ice cream!

Numbers

Numbers are an abstraction too. What do these objects have in common?

They are each a collection of 2 things.

Numbers are an abstraction of collections of things. Most of us learn about numbers when we are very young. We're good at abstraction even if we don't realise it.

Inverses

Inverses are how we undo things in maths. Negative numbers are a type of inverse of positive numbers.

For example, if you add 5 to a number and then add -5, you get back to where you started. Have a look at the diagram on the right to check this out. You will also see that dividing by a number is the inverse of multiplying by it.

Other inverses include:

- walking 5 steps right and then 5 steps left
- going up is an inverse of going down

Shapes

Shapes are an abstraction of everyday objects. For instance, calling a window a square is a way of turning a real thing (the window) into an abstract concept (a square).

We can also build shapes from lines using just our imagination. Sometimes we give shapes names because we talk about them a lot and want other people to know what we are referring to. What would you call the last shape we see in the diagram on the right?

Number Circles

Numbers don't have to be written as a line. They can also go in a circle, like on a clock.

The diagram on the right shows a 12-hour clock.

But what if we made a 10-hour clock instead? Would we have 20 hours in a day instead of 24?

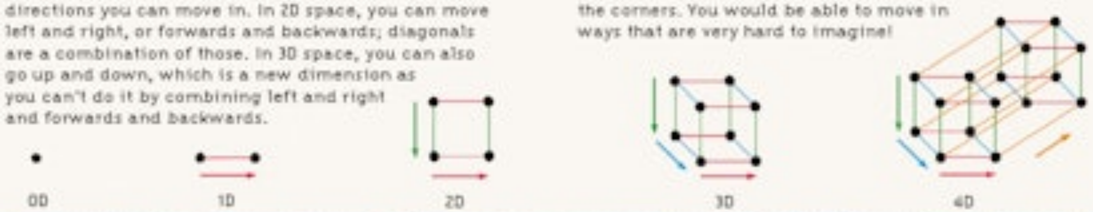
If we did that, we would need to make the hours longer or the days shorter.

Which would you prefer?

Dimensions

Squares and triangles are 2-dimensional (2D), but shapes can have lower dimensions... or higher ones! Dimensions of space are completely different directions you can move in. In 2D space, you can move left and right, or forwards and backwards; diagonals are a combination of those. In 3D space, you can also go up and down, which is a new dimension as you can't do it by combining left and right and forwards and backwards.

- A point is zero dimensional. You can't move anywhere!
- A line is 1D. You can go forwards and backwards.
- A square is 2D as it has a whole new direction – side to side.
- A cube is 3D as you can also go up and down.
- A 4D cube is made from two 3D cubes joined at the corners. You would be able to move in ways that are very hard to imagine!



Number Grids

Numbers can also be written as a grid, like a chessboard. We might want to label one direction with letters so that we don't get confused between columns and rows. This is especially useful when we use coordinates on maps, so we know where we are or where we need to go.

On this chessboard, the white knight is in square E5. Naming the square like this is a bit like naming a 2D number. What would you call the square the black king is in?

Number Lines

There are many ways to arrange numbers. Organising them in a line is just one of them. There is no biggest number so the number line never ends! Numbers keep getting bigger forever – we can say there are infinitely many of them. Negative numbers also keep going forever in the opposite direction. The number line below has no beginning and no end, even though it stops at the edge of the physical page.

Infinity

Infinity is an abstract idea representing something bigger than ordinary numbers, which are all finite. This number line has infinitely many numbers on it, but they're all finite, so infinity is not a point on this line.

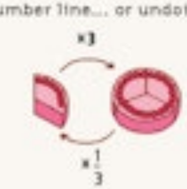
Fractions

A fraction represents part of a whole. Fractions come from thinking about dividing things up...

...and we write them like this:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$

We can also find fractions by filling in the spaces between numbers on the number line... or undoing multiplication.



As the bottom number of a fraction gets bigger, the fraction gets smaller. This is because we're inverting the process of multiplication by a very large number, so we need to make things very small again. The bottom number can get bigger forever so fractions can get smaller forever too!

Patterns in Numbers

Numbers produce patterns in all sorts of ways. Here we can see three number grids with different patterns marked in coloured boxes. Do you recognise any of these? Can you make any more patterns of your own?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counting in twos

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counting in threes

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counting in nines

Latin Squares

Latin squares are a pattern we can make with numbers or with anything else! We could use colours, shapes, patterns or objects. A Latin square is when there's only one of each thing in any row and any column. Here is a 5x5 Latin square. Do you remember seeing one similar to this on Molly's adventure?

On the right is a 12x12 Latin square made of numbers. It shows how time works on a 12 hour clock. Can you use the Latin square to work out the following?

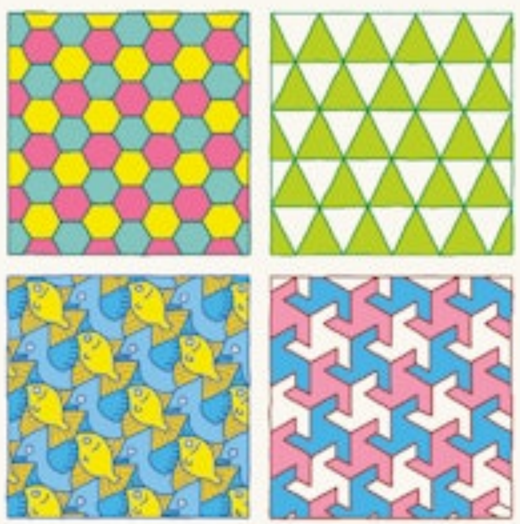
- 3 hours past 6 o'clock
- 6 hours past 11 o'clock
- 8 hours past 5 o'clock

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

Patterns in Shapes

Some shapes fit together to make patterns. This is known as tessellation, or tiling the plane. Triangles, squares and hexagons fit together well with themselves. But so can more complicated shapes if you try hard enough!

If shapes fit perfectly together like a jigsaw, this is known as a tessellation. Here are some examples:



Patterns in Nature

Patterns appear any time we see the same thing happening over and over again. This happens a lot in nature, including with flowers and their repeating petals:

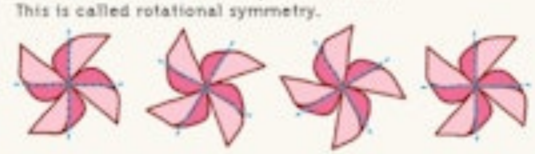
Patterns can appear on animals:

People use patterns to make things like wallpaper:

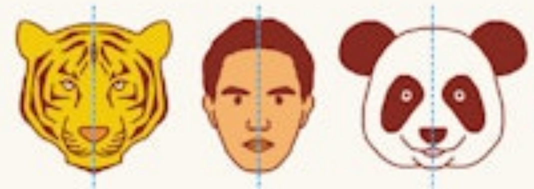
Symmetry

Symmetry is when a shape looks the same as itself when we turn it round, flip it over or possibly by all sorts of other transformations.

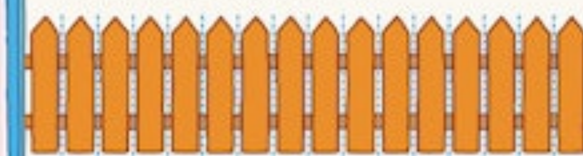
Windmills look the same if you turn them.
This is called rotational symmetry.



These faces look the same on both sides.
This is called reflectional symmetry.



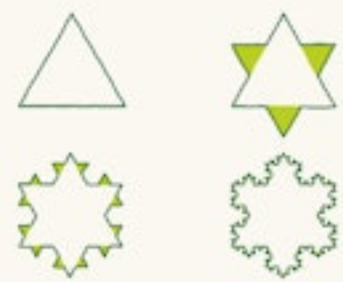
Some things look the same if we shift our view sideways.
This is called translational symmetry.



Can you think of any other ways something could be symmetrical? There are many more!

Fractals

Some things look the same when we zoom in on them and examine them more closely. The diagrams below show how to construct a fractal snowflake – at every step, you add a triangle to each side of the shape.

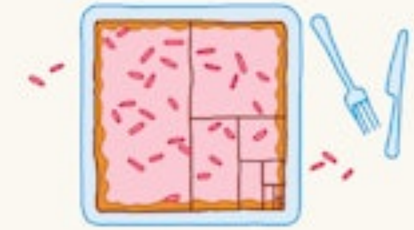


Fractals have a form of self-symmetry. You can zoom in on fractals forever. They have infinite amounts of infinitely small detail.

Paradoxes

Sometimes our attempts to use maths to think clearly create weird contradictions called paradoxes.

For example, infinity is a strange idea, and can lead us into paradoxes if we're not careful. Let's imagine eating a cake. What if you eat half of it, then half of what's left, and half of what's left of that, forever? Does that mean you'll never finish your cake?



The impossible staircase we saw in Molly's adventure is a bit like a visual paradox. It's impossible and possible at the same time, just in different dimensions.

Paradoxes sometimes come from sentences that sound logical but loop back on themselves in weird ways. What if I say, "I'm lying"? Am I lying... or not? It's impossible both ways!

Time Travel

Watch out if you ever find a time machine or a wormhole, because travelling in time can cause paradoxes – especially if you meet your past self.

What if you go back to yesterday, meet yesterday-you, and cut their hair? What will your hair be like when you travel back to today?